AN EXPERIMENTAL STUDY OF THE FLOW OF A THIN LAYER OF LIQUID ON THE SURFACE OF A ROTATING CONE

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Equations (2.3), (2.9), and (2.10) of [1] describe the flow of a thin layer of liquid on the surface of a rotating body of revolution. We use the symbols of [1] and put q = 0 and j = 0 to get for the case of axially symmetric flow that

$$Q = \frac{Q_0}{r(x)}, \qquad \frac{dv_2}{dx} = -\frac{r'}{r} v_2 - 2\omega r' - \frac{\lambda}{8Q} v v_2, \tag{1}$$

$$\frac{dh}{dx} = \frac{\omega^2 r r' + 2\omega v_2 r' + v_2^2 r'/r - QQ'/h^2 - \lambda v Q/8h^2}{f - Q^2/h^3}, \tag{2}$$

$$Q = v_1 h,$$

$$f = (\omega^2 r + 2\omega v_2 + v_2^2/r)\sqrt{1-(r')^2}, \quad Q_0 = \text{const}. \tag{3}$$

We eliminate Q from (1) and (2) to get a system of two ordinary differential equations that are easily integrated numerically provided that v_2 and h are known at some point $x = x_0$.

The critical depth is defined by $fh_*^3 = Q^2$ for a given Q; if $h > h_*$, the flow is subcritical and vice versa.

The dimensionless resistance coefficient λ appears in (1) and (2); this is best derived for turbulent flow from the generalized relation for C [2] in the case of open flow, which was derived via the semi-empirical theory of turbulence. This relation incorporates the roughness ε, the viscosity ν, and the slope i of the bed,

$$C = 20 \lg \frac{h}{\varepsilon + 0.385\nu/\sqrt{ghi}}, \tag{4}$$

provided that it is correct to replace the hydraulic radius by the depth h of the flow. Formula (4) applies for hydraulically smooth walls, for completely rough walls, and for the transition region, i.e., for the entire region of turbulent flow. From

$$C = \sqrt{8g/\lambda}, \qquad v = \sqrt{8ghi/\lambda}$$

we readily transform (4) to

$$\left(\frac{8}{\lambda}\right)^{1/2} = -\frac{20}{\sqrt{g}} \lg\left(\frac{\varepsilon}{h} + \frac{0.385}{\sqrt{1/_8\lambda}R}\right) \quad \left(R = \frac{vh}{\nu}\right). \tag{5}$$

From laminar flow, $\lambda = 24/R$, which follows from a consideration of laminar flow of a viscous liquid with a free surface.

If a flow of liquid is admitted to the surface of a rotating cone in such a way that $v_2 = 0$ at the point of admission, we can measure the depth h at this point to get the necessary boundary conditions for (1) and (2). The constant $Q_0 = Q_1/2\pi$, in which Q_1 is the flow rate.

For a cylindrical tube (r' = 0), system (1)-(3) becomes

$$\frac{dv_2}{dx} = -\frac{\lambda}{8Q} v v_2,$$

$$\frac{dh}{dx} = -\frac{\lambda v Q}{8h^2(\omega^2 r - 2\omega v_2 + v_2^2/r - Q^2/h^3)}, \quad Q = \text{const}. \tag{6}$$

If we consider the spiral flow in a thin layer that satisfies the boundary condition, we get $v_2 \equiv 0$, and so the shape of the free surface is defined by

$$dh/dx = -{}^1/_8\lambda Q^2/(\omega^2 r h^3 - Q^2). \tag{7}$$

This shows that r' = 0 allows only two forms for the free surface (subcritical and supercritical), the critical depth being given by

$$h_* = ({}^1/_4\, Q^2{}_1/\pi^2\, r^3\, \omega^2)^{1/3} = \text{const}.$$

In the supercritical state we expect a continuous reduction in h, while in the subcritical one we expect a continuous increase.

Equation (7) coincides with the hydraulic equation for steady-state motion of water in a prismatic bed of zero slope [3], with the centrifugal acceleration $\omega^2 r$ replaced by the force of gravity g; we there-

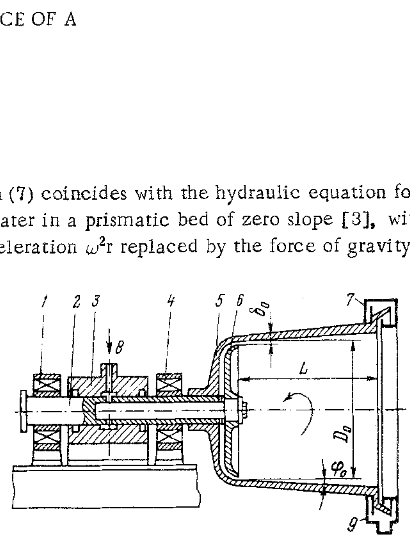

Fig. 1

fore do not need to repeat the argument of [3], and we use the result from hydraulics: the critical depth is set up at the onset when the flow moves in a bed of zero slope.

Figure 1 shows the apparatus for examining such flows on a rotating cone. Shaft 2 bears conical drum 5 in bearings 1 and 4, the shaft being belt-driven from a dc motor whose speed is controlled with a rheostat. The water passes through gland 3 and slot 8 into the hollow shaft. The water then passes through holes at the point of attachment of cone 5 to shaft 2 into the slot formed by disk 6 (δ_0 = 1 mm). The outgoing water is collected in a ring 7 and passes to a measuring tank.

Figure 2 shows the device used to measure the thickness of the layer. Needle 2 is held in holder 3, which can move relative to body 6, which itself may be turned by mechanism 7 relative to shaft 5. Holder 3 at point B is in contact with the knife of displacement indicator 8, which is mounted on the frame.

Needle 2 is joined by wire 4 to a device that indicates contact with the liquid, the circuit being provided by the metal cone and the needle.

The marked turbulence causes the free surface to fluctuate considerably, and, in general $h = h_1(t)$ is a random function; a mean h has therefore to be determined, e.g., the depth such that the needles spend half of a given interval T in the liquid and half in the air. Contact with the liquid unbalances an ac bridge containing the needle in one arm, the bridge operating at N_0 = 15 kHz. The voltage across the measuring diagonal is used to produce pulses, which are counted. The time of contact is then defined by the number of pulses recorded in time T to within one period of the sine wave. The reading is determined by N_0 if the needle touches the cone or is completely immersed in the liquid. The needle is adjusted to give a count $N = N_0/2$ in order to determine h. Each measurement was made with Q_1 = constant and n = constant, in which Q_1 is the volume flow rate and n is the speed of rotation. The flow rate is measured volumetrically, while n is measured to 2% with a tachometer.

The system is first adjusted to touch the conical surface with the cone at rest, the reading k_0 of the indicator being taken. Measurements were made at intervals of 20 mm along the cone.

Reading k_1 corresponds to the mean depth at steady Q_1 and n, so

$$h = (k_1 - k_0)\; l_1/l, \tag{8}$$

in which l_1 is the distance from the point to the axis of rotation of the holder and l is the distance from that axis to the point of contact with the knife (Fig. 2).

The error of measurement may be estimated from (8). The relative errors in l and l_1 do not exceed 0.1%, while the relative error

$x=$	15	35	55	75	95	115	135	155	175	195	211
$h'=$	1.850	1.820	1.830	1.690	1.50	1.380	1.490	1.450	1.270	1.090	0.675
$h=$	1.613	1.583	1.550	1.514	1.476	1.433	1.382	1.322	1.247	1.147	0.988
$\delta=$	12.8	13	15.3	10.4	1.6	3.84	7.25	8.85	1.80	5.22	46.5
$h'=$	0.745	0.772	0.672	0.655	0.561	0.595	0.630	0.592	0.600	0.565	0.343
$h=$	0.650	0.527	0.518	0.515	0.512	0.508	0.505	0.501	0.497	0.494	0.491
$\delta=$	12.7	31.8	22.9	21.4	8.74	14.6	19.8	15.1	17.2	12.5	39.2

in the counting system does not exceed 2%; the relative error in h also includes the relative error of the indicator. The relative error in de-

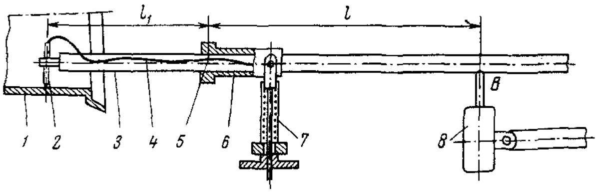

Fig. 2

termining k_0 consists of that in positioning the cone relative to the axis and that of the indicator. The absolute magnitudes of these two errors are 0.015 and 0.002 mm, and so the relative error in h is

$$\eta = 0.021 + 0.019\,/\,(k_1 - k_0),$$

in which $k_1 - k_0$ ranges from 0.2 to 0.7 mm. The η for $k_1 - k_0$ of 0.2, 0.45, and 0.7 mm are 11.6, 6.3, and 4.9%.

The measurements were made for several conditions. Let x be the distance along the cone from disc 6 (Fig. 1); D_0 = 218 mm and the length L = 215 mm. The surface finish of the cone corresponds to ε on Al'tshul's scale [2] from $2 \cdot 10^{-4}$ to $2 \cdot 10^{-3}$ cm. Calculation for the free surface on a cylinder ($\varphi = 0°$) reduces, from (7), to the integration

$$dx = \frac{8}{k}\left(\frac{\omega^2 r}{Q^2}h^3 - 1\right)dh \qquad (h = h_* \text{ for } x = L).$$

Equations (1)–(3) were used in calculating h as a function of position along the cone; these are compared with the measured h' (mm) (see table).

The values in the upper part are for $\varphi = 0°$, $Q_1 = 350$ cm^3/sec, and n = 600 rpm, while those in the lower part are for $\varphi = 3°$, $Q_1 = 500$ cm^3/sec, and n = 900 rpm. The discrepancy $\delta = |h' - h|/h'$ (in %) is largest near the start and end, where end-effects make themselves felt; the region of initial perturbation increases with Q_1, but the flow may be considered as axially symmetrical and steady at a distance of 12–14 cm, and here δ ranges from 1 to 14%, in part because (1) and (2) are approximate [1].

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